

# CONSISTENT NON-LINEAR STRUCTURAL ANALYSIS OF LONG-SPAN BRIDGES

Dorian JANJIC<sup>1</sup> and Harald SORSKY<sup>2</sup>

**ABSTRACT:** Most bridge structures are analysed using linear elastic theory. Results from such an analysis are usually an adequate basis for design since non-linear effects are small and can be accounted for by safety factors or even be ignored. Some non-linear effects are also taken into account to some degree on a design check level. However, during the analysis of long-span bridges, especially in cable suspended bridges, non-linear effects often reach magnitudes that make it imperative to include them into the analysis of such bridge structures. Much research has been produced on this topic, especially on cable-stayed and suspension bridges. However, in order to make this research accessible for practising engineers, a number of steps still had to be taken. This paper presents the results of a research project which investigated the theoretical background of non-linear structural bridge analysis. Generalisation of existing literature was complimented by further research to find ways of implementing non-linear procedures into a general bridge design software package suitable for use in a practical bridge design environment. The theoretical background of the application is outlined and the implementation into the bridge design software is described briefly. Practical application examples are given to illustrate the principles described in this paper.

**KEYWORDS:** cable-sagging, P-delta, wind buffeting, aero elastic damping, time effects, optimisation, erection control

## 1. INTRODUCTION

During the analysis of long span bridges, especially in cable suspended bridges, non-linear effects make it imperative to include them into the analysis. The basic problem of such analysis is how to combine different non-linear effects in consistent non-linear structural analysis. Time-effects have to be coupled with continuous change of structural system, cable-sagging, P-delta effects and even large displacement theory. The special solution has been found for optimisation problem for non-linear structures.

Within software package RM2006 [1], TDV-Austria, is this problem successfully solved. Based on implemented novel theoretical solution TDV software has been used for structural analysis of long-span bridges.

In this paper the theoretical background is outlined [2]. The representative application example for consistent non-linear structural analysis is Sutong bridge, part of the Suzhou-Nantong Yangtze River Bridge Project is located in the southeast of the Jiangsu Province in China.

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<sup>1</sup> *Managing Director, TDV GmbH, Austria*

<sup>2</sup> *Project Engineer, TDV GmbH, Austria*

## 2. CONSIDERING NON-LINEAR STRUCTURAL BEHAVIOUR

Cable-sagging, p-delta effects and large displacement theory are implemented within overall Newton/Raphson iteration procedure [3]. The implementation allows taking contact problems and non-linear material behaviour in the same procedure.

## 3. CONSIDERING TIME EFFECTS -

In practical engineering, time effects (creep, shrinkage and relaxation) must be considered in the analysis [4]. The occurrence of time dependent plastic strain is a material property of concrete. Total plastic strain consists of creep plastic strain and shrinkage plastic strain. Concrete creep models are generally defined by separated creep factors for each stress increment in the loading history. These creep factors depend of the loading time and of other factors like concrete quality, environment (humidity, temperature,...), section properties etc. The numerical solution is accomplished step by stepping in the time domain. Let time  $t_1$  corresponds to the start and time  $t_2$  to the end of a given time step. It can be assumed that complete load/response history is known (including all stress increments) from time equal to zero up to the start ( $t_1$ ) of the time step. Basic unknowns are stresses and strains at the end of time step ( $t_2$ ).

$$\varepsilon^2 = \varepsilon^1 + \Delta\varepsilon \quad , \quad \Delta\varepsilon = \int_{t_1}^{t_2} \frac{\partial \varepsilon}{\partial t} \cdot dt \quad (1)$$

$$\sigma^2 = \sigma^1 + \Delta\sigma \quad , \quad \Delta\sigma = \int_{t_1}^{t_2} \frac{\partial \sigma}{\partial t} \cdot dt \quad (2)$$

The total strain increment  $\Delta\varepsilon$  can be divided into elastic and plastic part. The solution is to iterate the corresponding plastic strain due to the additional elastic strain increment within a general Newton/Raphson procedure.

## 4. CONSIDERING CONTINUOUS CHANGE OF STRUCTURAL SYSTEMS

Continuous change of structural systems is another major reason for getting non-linear optimisation problems. To understand the physical reason, a very simple example of using a temporary cable in the construction schedule is shown in Figure 1. This non-linearity has a different nature to the time-effects non-linearity but it can be treated with the same optimisation method.

Bending moments on the main girder before closure (with temporary primary support active) and after closure on the main girder due to temporary support removing are presented in Figure 1.

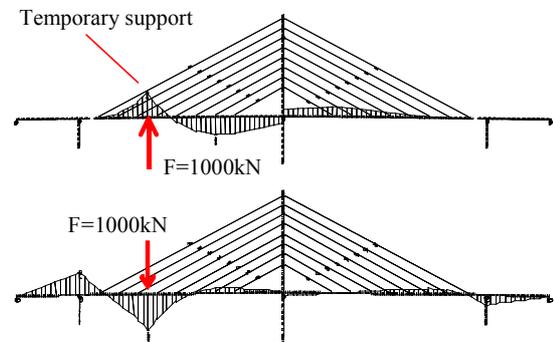


Figure 1. Bending moments on the main girder.

## 5. NUMERICAL ANALYSIS

The mathematical model of a structure consists of “General Properties” describing physical parameters like the material behaviour, of the “Geometric Model”, describing the alignment of the structure in space as well as the cross-section geometry, and the “Time Model”, describing the construction sequence and the structural behaviour during life-time [5].

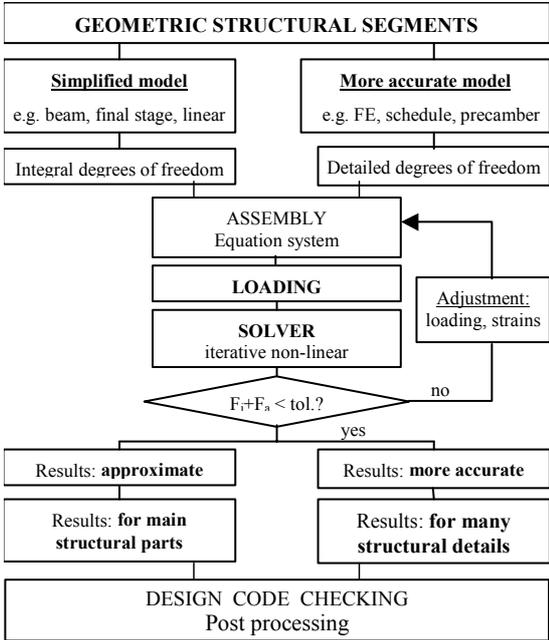
Once the geometric model has been established, the model data are passed to the central analysis unit. The time domain is considered from the very beginning by establishing a global time axis mirroring the time from the construction start during erection time and operation time to infinity.

The different structural parts are activated and the loading is applied at the respective points in time corresponding to the construction process on site. The time dependent effects occurring in the intervals (creep and shrinkage) are fully considered on the correct structural system.

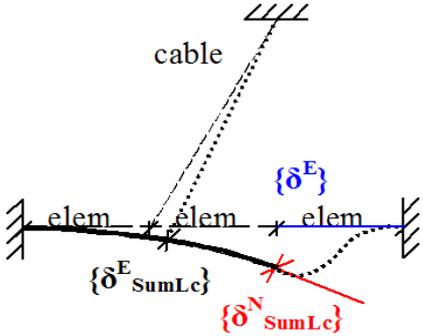
However, the "Schedule" is not only a reflection of the construction process on site, but a global framework defining the scheduled behaviour of the structure as well as the required investigations, optimisations and proof checks for any intermediate state during construction, and for the final stage after completion and at infinity. Variations of the schedule – e.g. in order to find the most economic proceeding or to adapt it to unforeseen events like time delays – can be easily performed by modifying or adding "schedule actions" at the appropriate point in time.

**6. PRE-CAMBER, ERECTION CONTROL**

The main purpose of the RM2006 erection and geometry control module [6] is to accurately control the position of the segments in a linear or non-linear bridge structure built with using the stage-by-stage construction method. The structural geometry of the segments and cables is defined by structural model and the individual shape of the segment elements and stress free lengths of the cables. For the 1st iteration of the calculation, the "Erection Control" procedure uses the pre-deformed structure, i.e. the deflection line taken from a previous calculation.



**Figure 2. Flowchart – Numerical Analysis.**



**Figure 3. Automatic kink correction in erection control mode.**

This enables both, linear calculation or a real 3rd order theory analysis. Structural assembling with the option "Automatic Kink Correction" is commonly used in this mode - where each newly activated element is fully constrained by a face-to-face connection with the currently active displaced structure.

Using the erection control facility, the stress free fabrication shape is applied as a load to the structure. In general, the application of the stress free fabrication shape of a segment to the currently active structure will produce forces, stresses and deformations. Since the stress free shape is applied as a loading, the user can, using this device, control and optimise the forces in the structure.

## 7. OPTIMISATION

Special optimisation procedures are necessary for standard bridge design process. The AddCon Method (The Additional Constraint Method) is a novel solution for optimisation problems in structural engineering [7].

If structural response is not linear, the optimisation problem is non-linear from the very beginning. In practical cases this non-linearity is not too far away from a linear solution. Design experience shows that non-linear effects are usually within 20% of linear solution. This is the same order of magnitude as the non-linearity due to time effects [8]. Again, these effects can be treated with the same method de-scribed above. With a mild non-linearity grade we cover almost all problems.

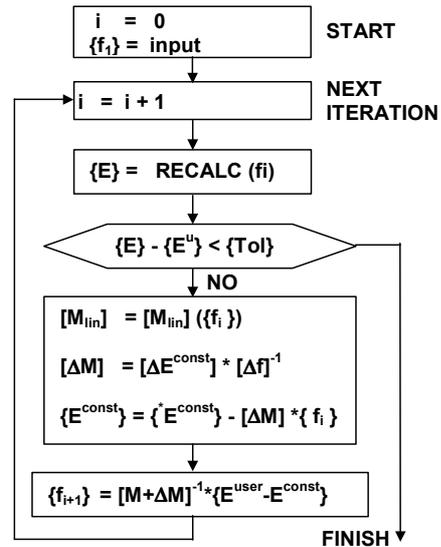


Figure 4. Flowchart – Iterative search for solution of Optimisation

## 8. WIND BUFFETING ANALYSIS

The wind related functions [9] of RM2006 match nearly all needs for the design of long-span bridges. Arbitrary complicated wind profiles with varying wind speed and turbulence intensity are easily defined. Together with the cross-section related shape factor diagrams defining the dependency of the drag- lift- and moment coefficients on the attack angle of the wind impact, these wind profiles allow a comprehensive wind buffeting analysis taking into account the varying along-wind and lateral forces of gusty wind events.

The structural wind buffeting calculation is performed in the modal space and in the frequency domain. It includes aerodynamic damping and stiffness effects [10] due to structural movement caused by the wind flow. All computations are based on the tangential stiffness of the structure at a given point in time – the structure under permanent loading and mean wind – allowing for including all prior non-linear effects.

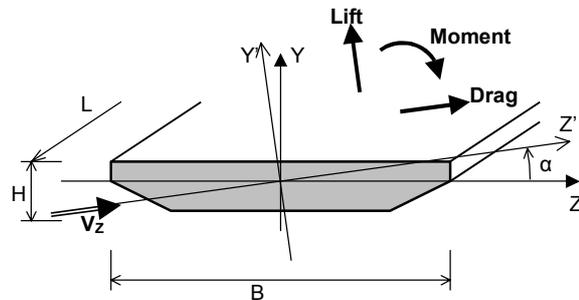
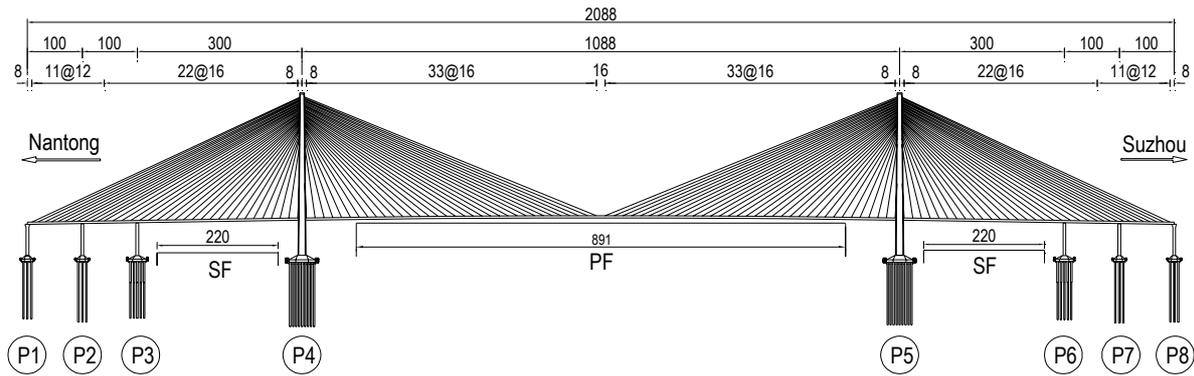


Figure 5. Velocity of the structure and wind fluctuation

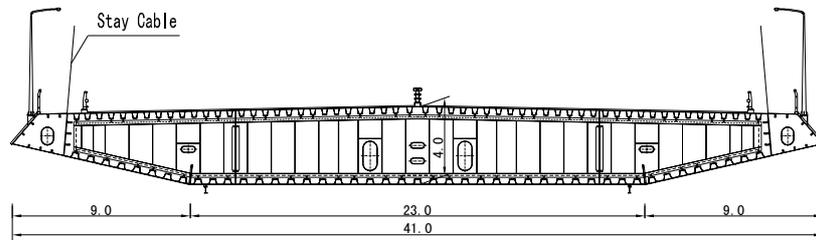
## 9. APPLICATION EXAMPLE – SUTONG BRIDGE

The Suzhou-Nantong Yangtze River Bridge Project [11] is located in the southeast of the Jiangsu Province in China. The total length of the bridge portion in this link is about 8.2 km. The design tasks of the project are carried out by China Highway Planning and Design Institute (HPDI) Consultants, Inc. in cooperation with Jiangsu Provincial Communication Planning & Design Institute, and the Architectural Design & Research Institute of Tongji University. Main cable stayed bridge is a double-plane twin-pylon bridge with a continuous span arrangement of (100+100+300+1088+300+100+100) m, as shown in Figure 6. Two auxiliary piers and one transitional pier are erected in each side span. The main span of the bridge is 1088 m, which is the longest main cable-stayed bridge span at present.



**Figure 6. Span Arrangement (unit: m)**

The bridge girder is a streamlined closed flat steel box girder. The total width including wind fairing is 41.0 m accommodating dual 8 traffic lanes. The cross-section height is 4.0 m. The steel box is generally stiffened in the longitudinal direction with closed steel troughs. The standard cross-section of the girder is illustrated in Figure 7.



**Figure 7. Cross Section of the Girder (unit: m)**

The inverted concrete Y-shaped pylons are about 300 m in height. The stay-cables are anchored inside steel boxes fixed to the concrete by shear studs at the pylon top. The stay cables are arranged in double inclined cable planes with standard spacing of 16 m in the central span and 12 m near the ends of the back spans along the girder. To reduce the effect of wind loads, the cable stay systems are made of the parallel wire strand consisting of 7 mm wires, each with a cross sectional area of 38.48 mm<sup>2</sup>. The selected permanent connection between the girder and the pylons is accomplished by nonlinear dampers as used for the Great Belt East Bridge in Denmark.

The RM2006 program developed by TDV, Austria, has been used for the global analysis of the SuTong cable-stayed bridge in the detailed design. The structural modelling keeps accordance with the planned construction schemes. Each of the stay-cables was divided into 8 sub-elements to consider cable-sag effects rather than approximating this effect by using effective module of elasticity. Other interacting non-linear effects such as P-delta effect, large displacements and shear displacements were also considered in the calculation. Creep and shrinkage effects were calculated according to the CEB/FIP 90 code. The flexibility of the pylon foundations was modelled with spring elements. The connection between the girder and both pylons were treated as nonlinear static spring elements with a gap value of 750 mm and a linear stiffness of 100 MN/m.

The forward analysis with the ADDCON method implemented in RM2006 was employed for all erection stages to achieve the final situation as described above according to the construction schedules of the designer. All temporary supports, tie-downs, and movements of derricks for construction, temporary loading, and permanent loading were included in the model at various stages. The equivalent static wind actions from different directions were also investigated at the most detrimental construction stages such as the maximum double cantilever, the maximum single

cantilever and the completed bridge. The pre-camber calculation of all construction stages was computed by RM2006 automatically. The third-order effect of pre-camber shapes apart from the design elevation of the deck was also considered approximately.

## 10. CONCLUSION

Using a finite difference in the time domain and Newton/Raphson for structural non-linearities the presented algorithm includes coupled effects of the creep, shrinkage and steel relaxation within overall structural non-linear analysis. Cable sagging, p-delta effects, large displacements and contact problems are combined with long term effects within consistent analysis. The proposed method for the numerical analysis is generally suitable for the investigation of all kinds of bridges with reinforced concrete, pre-stressed concrete or composite sections. Figure 8 shows the bending moment envelope in deck for the Sutong Bridge, China (Project engineers: HPDI, Beijing). With main span of 1088m this is the longest cable stay bridge at present.

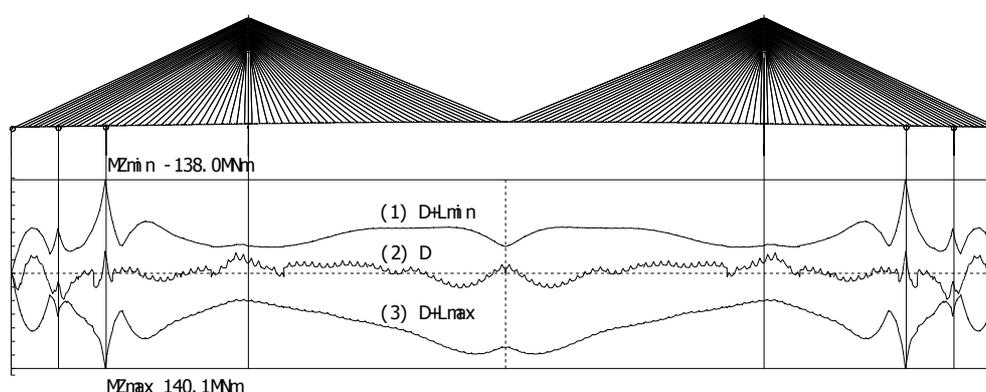


Figure 8. Bending moment envelop in deck

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